International Journal of Theoretical Physics, Vol. 5, No. 1 (1972), pp. 55–70

Nonlocal Theory of Single Pion Photoproduction

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Received: 13 *April* 1971

Abstract

A nonlocal interaction theory is formulated for photoproduction processes by introducing a form function. An effective nonlocal, Lorentz invariant and gauge invariant Lagrangian density for the four-field interaction that gives rise to a production amplitude is constructed. The general structure of the form function is investigated by using some restrictions of the form function. For low energy π^0 photoproduction an explicit form of the Fourier component of the form function is obtained. The physical model of the present formalism is to assume, similar to the strong absorption model, that the system in intermediate states is confined in a finite domain of space. For π^0 production the linear dimension of this domain is obtained to be $\Omega_0 = 3{\cdot}88F$. It is important to observe that the extent of electromagnetic distribution in a nucleon is also nearly the size of Ω_0. This is believed to be the reason that in low energy pion photoproduction the effects of electromagnetic structure of a nucleon are irrelevant. The unpolarized differential and total cross sections are calculated for π^0 production in helicity representation and the predictions are found to be in good agreement with experiments.

1. *Introduction*

The theoretical analysis of the photoproduction of mesons has been studied by many authors. A review of these developments will not be given here. We refer the reader to Donnachie and Shaw's paper (Donnachie & Shaw, 1966) for a systematic review of the developments within the frameworks of dispersion relation and isobar-exchange model.

In this paper we present an analysis of single pion photoproduction based on a nonlocal interaction theory. In this formalism, as shown in Section 2, a nonlocal interaction form function is introduced into the transition matrix. Applying the Lorentz invariance, as shown in the Appendix, some restrictions of the form function are obtained. The general form of structure of the Fourier components of the form function are investigated by using these restrictions. The Fourier components of the form function represent the energy correlation functions between fields in intermediate states.

The physical model of this formalism is to assume that the system in intermediate states is confined in a finite domain of space Ω_0. In concept, the present model is in this respect similar to the strong absorption model (Frahn & Venter, 1963) in which one assumes the existence of an absorbing sphere and actually paramatizes the size of the absorbing sphere in calculations (Eisenberg *et al.*, 1966). The linear dimension of this domain in present calculation is taken to be the wave length corresponding to the mean energy of intermediate states. With this model the explicit forms of the Fourier components of the form function can be obtained for low energy π^0 photoproduction as shown in Section 3. The unpolarized differential and total cross sections for π^0 production are calculated in helicity representation and the comparisons with experimental data are given.

2. *Formalism*

The S-matrix for photoproduction is written as usual

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta(P_i - P_f) T_{fi} \tag{2.1}$$

where T is the transition matrix, and $P_i = k + p_1, P_f = q + p_2$. The quantities k, q, p_1 and p_2 are four-momenta of the incident photon, the outgoing meson, the initial nucleon, and the final nucleon respectively. For scalar meson photoproduction, the transition matrix is expressed in the present formalism as

$$T = \frac{-ieg_{\pi N}}{2} \int \frac{1}{4!} \sum_P M(x, x', x'', x''') \underline{\phi}(x) . [\bar{\psi}(x'), \underline{\tau} \gamma_5 Q_\mu \psi(x'')] \times A_\mu(x''') d^4 x d^4 x' d^4 x'' d^4 x''' \tag{2.2}$$

In equation (2.2) the $\sum_P$ indicates the sum over all permutations between x, x', x'', x'''. The four-vector operator Q_μ is a function of γ-matrix. Its structure depends on the photon energy. For photon energy of less than 0·5 Gev Q_μ is assumed to have a very simple form, namely $Q_\mu = \gamma_\mu$. However, when photon energy is greater than 0·5 Gev, the effects of the detailed electromagnetic structures of nucleon on photoproduction process are believed to become important. Then the operator Q_μ should contain the information of nucleonic electromagnetic structure and should be written as (Drell & Zachariasen, 1961)

$$Q_\mu^{p,n} = F_1^{p,n}((p_1 - p_2)^2) \gamma_\mu + iF_2^{p,n}((p_1 - p_2)^2) \sigma_{\mu\nu}(p_1 - p_2)_\nu \tag{2.3}$$

The function $M(x, x', x'', x''')$ is an interaction form function between fields. It describes the non-local interaction between fields in photoproduction processes. Generally, this function is taken to be a function of coordinates of the interacting fields. A general expression for this function is still unknown. However, if we assume that the Lorentz invariance holds within the domain of Ω_0 (Rédei, 1967), we can obtain some restrictions of the form function. As shown in the Appendix, if it is required that the energy-momentum is conserved under the translational transformation of space-time, one obtains a restriction

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_\mu{}^i} M(x', \ldots, x^n) = 0 \tag{2.4}$$

On the other hand, if we require that angular momentum is conserved under rotational transformation of space-time and for non-local interaction an integral symmetry of the energy-momentum tensor is satisfied, then one obtains another restriction

$$\sum_{i=1}^{n}\left(x_\mu{}^i\frac{\partial}{\partial x_\nu{}^i}-x_\nu{}^i\frac{\partial}{\partial x_\mu{}^i}\right)M(x',\ldots,x^n)=0 \tag{2.5}$$

Some interesting consequences of these restrictions, particularly restriction (2.4) which is generally true, on the form function is best seen by considering the Fourier transformation of the form function

$$M(x',\ldots,x^n)=\frac{1}{(2\pi)^{4n}}\int M(k',\ldots,k^n)\exp\left[i(k_\mu{}'x_\mu{}'+\cdots+k_\mu{}^n x_\mu{}^n)\right]\times$$
$$\times\, d^4k'\ldots d^4k^n \tag{2.6}$$

Substituting equation (2.6) into equation (2.4), one obtains in Fourier space

$$\sum_i k_\mu{}^i M(k',\ldots,k^n)=0 \tag{2.7}$$

Equation (2.7) implies that $k_\mu{}^n$ can be solved as a function of $k_\mu{}',\ldots,k_\mu^{n-1}$. Furthermore, if one recalls that $x\delta(x)=0$ the Fourier components can be expressed in the following form

$$M(k',\ldots,k^n)=(2\pi)^4 M(k',\ldots,k^{n-1})\,\delta(k_\mu{}'+\cdots+k_\mu{}^n) \tag{2.8}$$

and equation (2.6) becomes

$$M(x',\ldots,x^n)=\frac{1}{(2\pi)^{4n-4}}\int M(k',\ldots,k^{n-1})$$
$$\times\prod_{j=1}^{(n-1)}\exp\left[ik_\mu{}^j(x_\mu{}^j-x_\mu{}^n\right)d^4k',\ldots,d^4k^{n-1} \tag{2.9}$$

Now if we let

$$M(k',\ldots,k^{n-1})=1 \tag{2.10}$$

equation (2.9) immediately reduces to

$$M(x',\ldots,x^n)=\prod_{i=1}^{(n-1)}\delta(x_\mu{}^i-x_\mu{}^n) \tag{2.11}$$

which is the local limit, i.e., by substituting equation (2.11) into equation (2.2) one obtains the conventional local interaction field theory. The implications of equations (2.8) and (2.10) on the general form of structure of the Fourier components of the form function are very essential. In the next section, a detailed structure of the form function will be developed for process of neutral pion photoproduction.

Let us now consider an observable, namely the differential photoproduction cross section. In helicity representation (Jacob & Wick, 1959), the unpolarized differential pion production cross section in the c.m. system can be written as

$$\frac{d\sigma}{d\Omega}=\frac{1}{6}\frac{1}{(8\pi)^2}\left(\frac{p'}{p}\frac{1}{S}\right)\sum_{(\lambda)}|f_{\lambda_2\lambda_q;\lambda_1\lambda_k}(\theta,\phi)|^2 \tag{2.12}$$

where the helicity amplitude can be expressed in partial wave expansion

$$f_{\lambda_2\lambda_q;\lambda_1\lambda_k}(\theta,\phi)=\sum_J (J+\tfrac{1}{2})\langle JM;\lambda_2\,\lambda_q|\,T^{(J)}\,|JM;\lambda_1\,\lambda_k\rangle \times \exp[i(\lambda-\mu)\phi]\,d^J_{\lambda\mu}(\theta) \tag{2.13}$$

In equation (2.12) the indices λ_1, λ_2, λ_k and λ_q stand for the helicities of the initial nucleon, the final nucleon, the incoming photon and the produced meson respectively. All other quantities are well defined in other places (Jackson & Hite, 1967). The matrix $T^{(J)}$ is a submatrix of the transition matrix T belonging to definite values of J. The matrix elements are taken between the asymptotic helicity states of the incoming photon and nucleon and the asymptotic outgoing meson and nucleon states. In the present formalism, one introduces an intermediate orthonormal complete set of state vectors $\{|\alpha\rangle\}$ describing the coupling states of interacting fields, and the submatrix element in equation (2.13) can be expanded

$$\langle JM;\lambda_2\,\lambda_q|\,T^{(J)}\,|JM;\lambda_1\,\lambda_k\rangle=\sum_{\alpha,\beta}\langle JM;\lambda_2\,\lambda_q|\,\alpha\rangle \times \langle\alpha|T^{(J)}|\,\beta\rangle\langle\beta|JM;\lambda_1\,\lambda_k\rangle \tag{2.14}$$

For low-energy photopion productions, we assume that the set $\{|\alpha\rangle\}$ can be represented by angular momentum representations $\{|JM;LS\rangle\}$ with channel spin S and relative orbital angular momentum L in Russell-Saunder's coupling. With this representation and the transformation matrix (Jacob & Wick, 1959)

$$\langle JM;\lambda_i\,\lambda_j|JM;LS\rangle=\left(\frac{2L+1}{2J+1}\right)^{1/2}\langle J\mu|LO;S\mu\rangle\langle S\mu|S_j-\lambda_j;S_i-\lambda_i\rangle \tag{2.15}$$

expression (2.14) can be rewritten as

$$\langle JM;\lambda_2\,\lambda_q|T^{(J)}|JM;\lambda_1\,\lambda_k\rangle=\sum_{LSL'S'}\left[\frac{(2L+1)(2L'+1)}{(2J+1)}\right]^{1/2} \times\langle J\lambda|LO;S\lambda\rangle\langle S\lambda|S_k-\lambda_k;S_1\,\lambda_1\rangle \times\langle J\mu|L'\,O;S'\,\mu\rangle\langle S'\,\mu|S_q-\lambda_q;S_2\,\lambda_2\rangle \times\langle JM;L'\,S'|T^{(J)}|JM;LS\rangle \tag{2.16}$$

where the quantities $\langle JM|j_1 m_1; j_2 m_2\rangle$ are the usual Clebsch–Gordon coefficients. The problem is now reduced to the calculations of the sub-matrix elements $\langle JM; L'S'|T^{(J)}|JM; LS\rangle$. As an example, the calculations will be performed for the case of neutral pion production in the next section.

3. *Neutral Pion Photoproduction*

In photoproduction processes, when photon energy is below 0·5 Gev the contribution from multipion production is negligible. However, if the photon energy is above 0·5 Gev, processes involving more than one neutral product can become important (Cambridge Bubble Chamber Group, 1966). In order to avoid these difficulties, we will here consider the single neutral pion production for $E_\gamma \leqslant 0{\cdot}5$ Gev, i.e. in the region of first resonance. In this process, there is no charge exchange and the proton is not excited. Thus, in the present model, one assumes that the essential functions of the proton in process are two-fold, namely to conserve the monenta and to provide the domain Ω_0. Furthermore, for low energy production as discussed in the previous section, the effects of the detailed electromagnetic structure of nucleon on the process are negligible. One can then make the following simplifications for the form function and the operator Q_μ

$$M(x, x', x'', x''') = M(x, x''')\,\delta(x' - x'') \tag{3.1a}$$

and

$$Q_\mu = \gamma_\mu \tag{3.1b}$$

and the transition matrix of equation (2.2) can be simplified as

$$T = (-ieg_{\pi N}) \int \frac{1}{2!} \sum_P M(x, x'')\,\phi^{(-)}(x)\,\bar{\psi}^{(-)}(x') \times \gamma_5\,\gamma_\mu\,\psi^{(+)}(x')\,A_\mu(x'')\,d^4x\,d^4x'\,d^4x'' \tag{3.2}$$

The field operators $\phi^{(-)}(x)$ and $A_\mu(x'')$ are respectively the π^0 creation operator and the photon 4-potential operator which annihilates the incoming photon. In the present formalism, these operators are expanded in terms of spherical waves describing states of definite angular momentum. For $\phi^{(-)}(x)$ one has (Muirhead, 1965)

$$\phi^{(-)}(x) = \left(\frac{1}{R}\right)^{1/2} \sum_{qlm} \frac{q}{(\epsilon)^{1/2}} j_l(qr)\, Y_l^{m*}(\hat{r})\,\alpha_l^{m\dagger}(q) \exp(i\epsilon t) \tag{3.3}$$

where the operator $\alpha_l^{m\dagger}(q)$ creates a π^0 with quantum numbers l and m for angular momentum and q for the linear momentum, and the function $j_l(qr)$ are the conventional spherical Bessel functions. The quantity R is the radius of a three-dimensional sphere such that the Bessel functions are normalized according to

$$\int_0^R j_l(qr)\,j_l(q'r)\,r^2\,dr = \frac{R}{2q^2}\,\delta_{q'q} \tag{3.4}$$

For the 4-potential, one has

$$A_\mu(x) = \sum_{\omega, j, M} C_{\omega jM}(A_{\omega jM})\mu \tag{3.5}$$

where the operator $C_{\omega jM}$ annihilates a photon of energy ω, angular momentum j and its z-component M. The spherical wave $(A_{\omega jM})_\mu$ describes photon state of definite angular momentum, energy and parity. The explicit form of $(A_{\omega jM})_\mu$ depends on the induced transition being due to electric 2^L-pole or magnetic 2^L-pole. For magnetic multipole transitions, which will be interesting in our case, $(A_{\omega jM})_\mu$ can be written (Akhiezer & Berestetski, 1965)

$$\underline{A}_{\omega jM} = \frac{1}{4\pi}\left(\frac{k}{R}\right)^{1/2} g_j(kr)\mathbf{Y}_{jM}^{(0)}(\hat{r})\exp(i\omega t) \tag{3.6a}$$

and

$$(A_{\omega jM})_0 = 0 \tag{3.6b}$$

The function $\mathbf{Y}_{jM}^{(0)}(\hat{r})$ is the vector spherical harmonic and can be expressed as

$$\mathbf{Y}_{jM}^{(0)}(\hat{r}) = \frac{1}{[j(j+1)]^{1/2}}\underline{L}Y_j{}^M(\hat{r}) \tag{3.7}$$

In equation (3.7), L is the relative orbital angular momentum with respect to the nucleon. The function $g_j(kr)$ is again a normalized spherical Bessel function in a three-dimensional sphere of radius R. Since the proton is not excited, it is adequate to use the ordinary plane wave expansion for the proton field. Then, by combining equations (3.2), (3.3) and (3.6a) and by a lengthy but straightforward calculation, the submatrix element in equation (2.16) can be written as

$$\langle JM;L'S'|T^{(J)}|JM;LS\rangle$$
$$= \frac{(eg_{\pi N})(2\pi)^3}{8\sqrt{(15)}}\mathscr{M}(k^*)\sum_{\substack{m_k,m,\\ m_2}}\langle JM|L_q M_q;\tfrac{1}{2}m_2\rangle\langle JM|L_k M_k;Sm_k+m_1\rangle$$
$$\times\langle Sm_k+m_1|1m_k;\tfrac{1}{2}m_1\rangle[T_{nf}(\theta)+T_f(\theta)] \tag{3.8}$$

The functions $T_{nf}(\theta)$ and $T_f(\theta)$ are nucleon spin non-flip and spin flip terms respectively and have the following expressions

$$T_{nf}(\theta) = \left[1 - \frac{|p_1||p_2|\cos\theta}{(E_1+M)(E_2+M)}\right]\delta m_2, m_1 \tag{3.9}$$

$$T_f(\theta) = \frac{|p_1||p_2|\sin\theta}{(E_1+M)(E_2+M)}\exp(i\phi)\,\delta m_2, -m_2 \tag{3.10}$$

where θ is the angle between incoming photon wave vector k and the outgoing pion momentum in c.m. system. The indices m_1 and m_2 are spin indices for nucleon before and after pion production. The function $\mathscr{M}(k^*)$ is written as

$$\mathscr{M}(k^*) = \int M(q,k^*) \frac{dq}{(q^2 - m_\pi^2)^{1/2}} \tag{3.11}$$

where k^* is the photon wave vector in c.m. system and $M(q,k^*)$ is the Fourier component of the form function $M(x,x'')$. It represents the energy correlation function between the photon and the neutral pion fields in intermediate states. According to equation (2.8), the function $M(q,k^*)$ can be written as

$$M(q,k^*) = 2\pi A(q)\,\delta(q-k^*) \tag{3.12}$$

where $A(q)$ describes approximately, in this calculation we exclude the proton energy, the energy amplitude in intermediate states and is an explicit function of the outgoing pion wave vector q. An intermediate state in this case is certainly not a stationary state of definite energy. Quantum mechanically, one can easily show that the energy amplitude function $A(q)$ for a decaying state of finite life time can be expressed as

$$A(q) = \frac{i}{2\pi} \frac{1}{q - q_0 + \dfrac{i\hbar\lambda}{2}} \tag{3.13}$$

where λ stands for the decay constant of the intermediate state and $\hbar\lambda$ is the width of the decaying intermediate states. The quantity q_0 is interpreted as the mean energy of the intermediate states with a spread of $\hbar\lambda$. In the present model, the whole system in intermediate states is assumed to be confined in a domain Ω_0. Thus the width of the decaying intermediate states can be written as

$$\Gamma = \hbar\lambda = \frac{h^2}{m_\pi \Omega_0^2} \tag{3.14}$$

Using equations (3.8) to (3.12), the submatrix element

$$\langle JM; L'S' | T^{(J)} | JM; LS \rangle$$

and hence the differential cross sections can be calculated.

Thus the entire theory has only two constants, namely q_0 and Ω_0. In order to determine the quantity q_0, we note the phenomenon (Källen, 1964) that the experimentally observed large maxima appearing in total cross sections for pion-nucleon scattering and photopion production processes are probably caused by the same intermediate resonant state. In pion-nucleon scattering the maximum in total cross section occurs at pion laboratory energy of 0·185 Gev. Thus, in the present calculations we shall assume that the quantity q_0 equals the total laboratory energy of pion at maximum scattering cross section, which is $q_0 = 0{\cdot}32$ Gev. The domain Ω_0

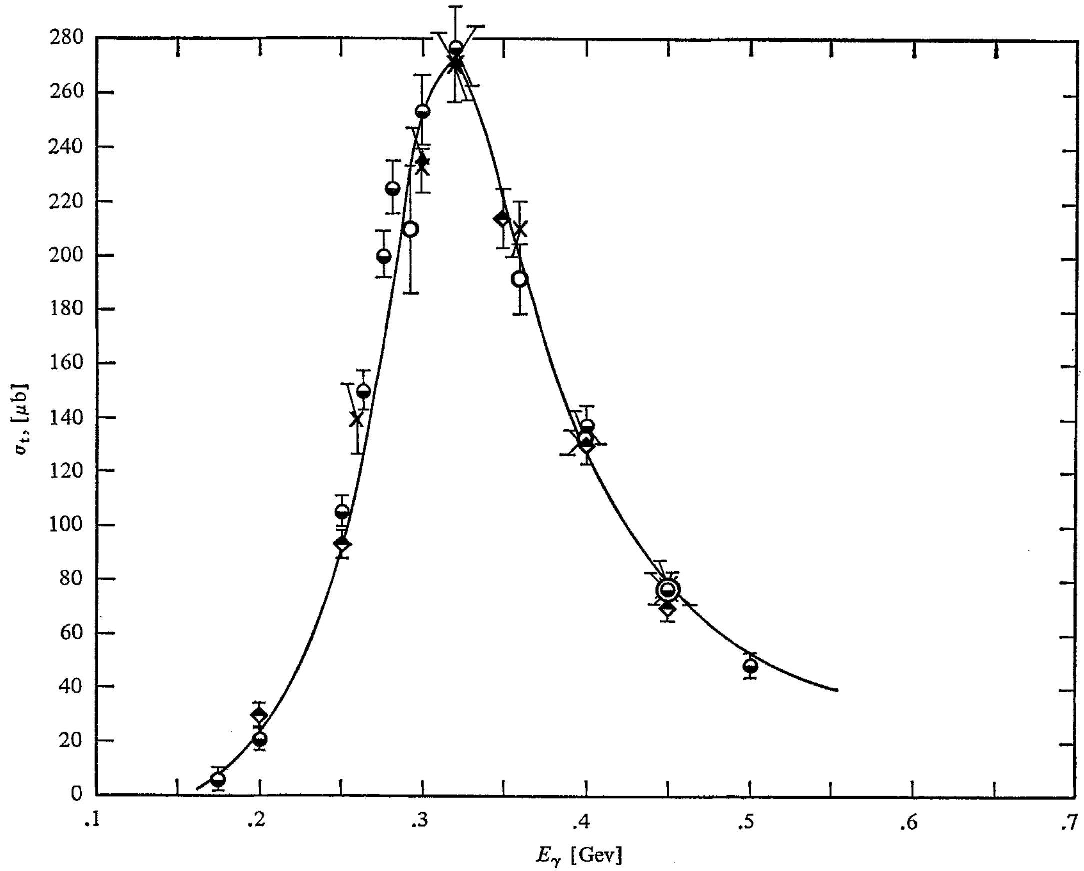

Figure 1.—Total cross-section of reaction $\gamma p \to p\pi^0$, E_γ is photon laboratory energy. Solid curve is the present theory. References: ⚲ (Oakley & Walker, 1955); ⚲ (Källen 1964); ⚲ (Goldschmidt-Clermont *et al.*, 1954); ⚲ (Walker, 1963); ⚲ (McDonald *et al.*, 1957).

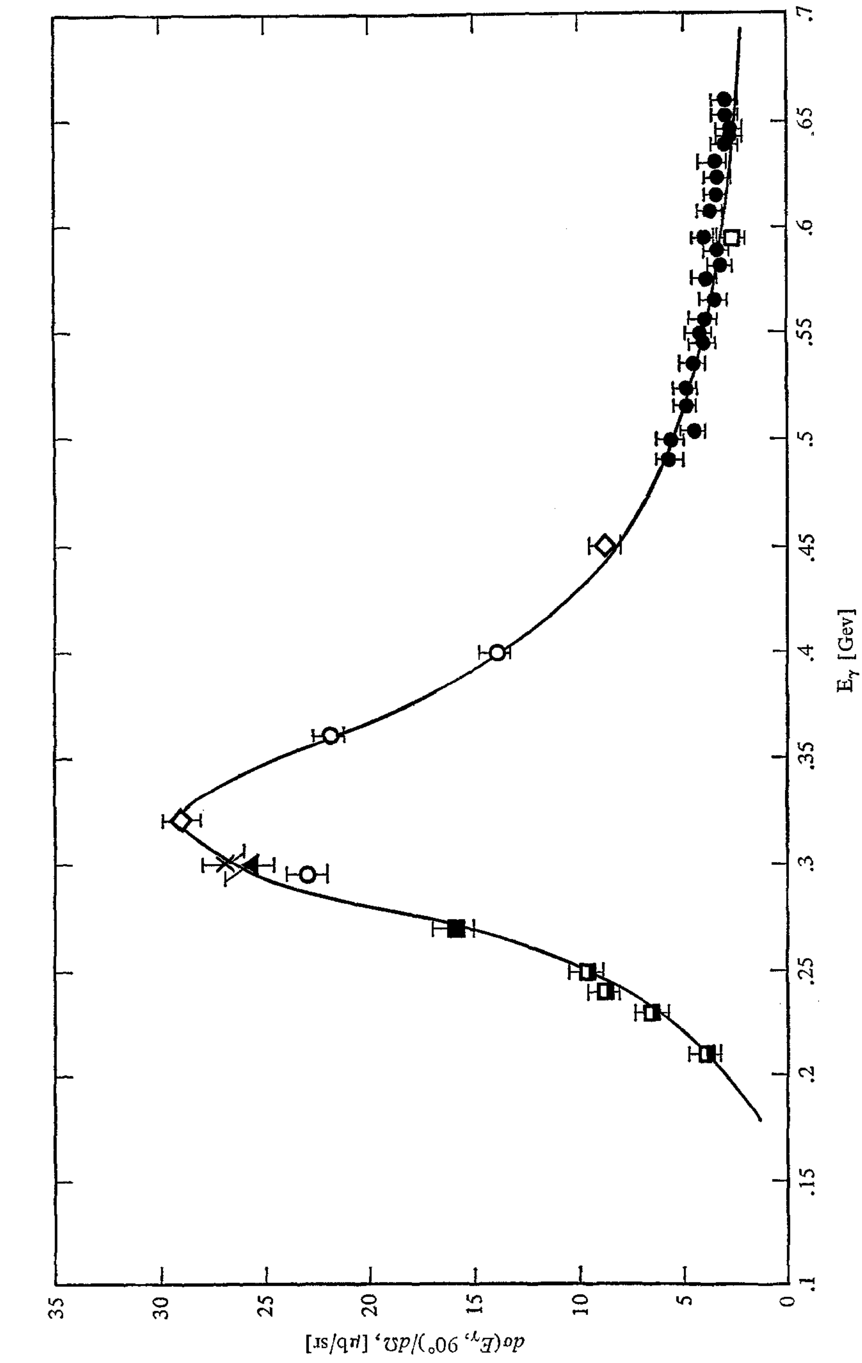

Figure 2.—The excitation curve for π^0 production at 90° c.m. Solid curve is the present theory. References: ⌶ (Donnachie & Shaw, 1966); ⌶ (DeWire *et al.*, 1958); ⌶ (Bacci *et al.*, 1967); ⌶ (Diebold, 1963).

is taken to be the wave length corresponding to q_0 which is $3{\cdot}88F$, where F stands for the unit of length in Fermis.

The photoproduction cross-sections for π^0 are calculated for magnetic dipole transition $M_{1+}^{(3)}$, and the differential cross-section when normalized at $\theta = 90°$ can be written as

$$\frac{d\sigma(E_\gamma, \theta)}{d\Omega} = \{[2 + 36F^2(\theta)] + [3 - 23F^2(\theta)]\sin^2\theta\}/(5 + 13f^2) \quad (3.15)$$

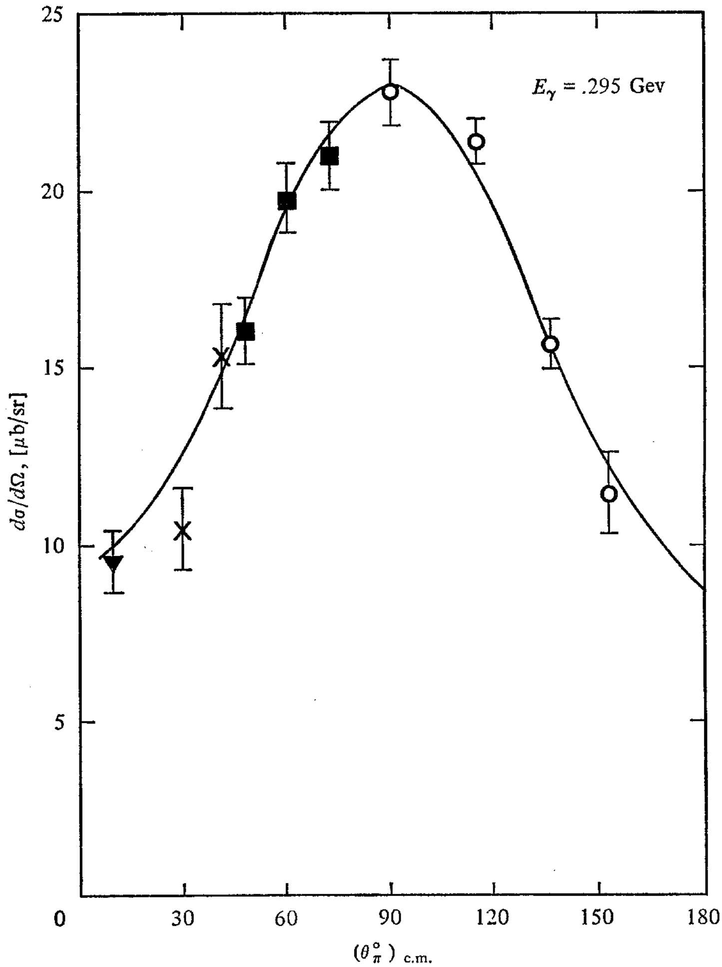

Figure 3.—Center-of-mass angular distributions for π^0 photoproduction at $E_\gamma = 0{\cdot}295$ Gev. Solid curve is the present theory. References: ⫯ (Berkelman & Waggoner, 1960); ⫯ (Highland & DeWire, 1963).

where

$$F(\theta)=\frac{f\sin\theta}{1-f\sin\theta} \tag{3.16}$$

$$f=\left[\frac{(1-X_1)(1-X_2)}{(1+X_1)(1+X_2)}\right]^{1/2} \tag{3.17}$$

and

$$X_1=[1+E_\gamma/0{\cdot}469]^{1/2}/(1+E_\gamma/0{\cdot}938) \tag{3.18}$$

$$X_2=[1+E_\gamma/0{\cdot}469]^{1/2}/(0{\cdot}9896+E_\gamma/0{\cdot}938) \tag{3.19}$$

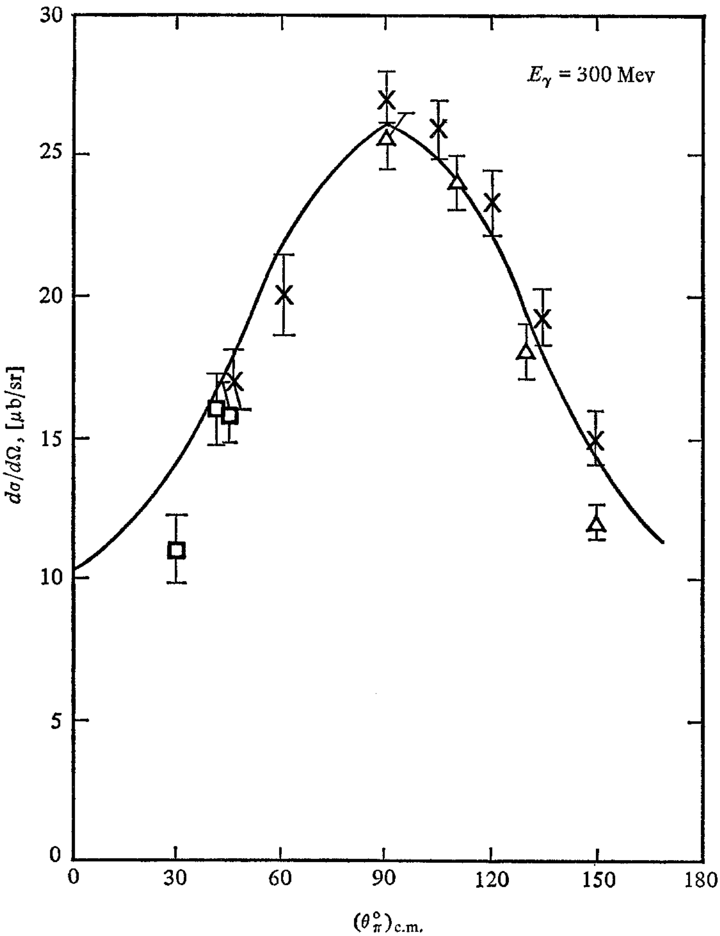

Figure 4.—Center-of-mass angular distributions for π^0 photoproduction at $E_\gamma = 0{\cdot}300$ Gev. Solid curve is the present theory. References: $\overline{\underline{\times}}$ (Walker *et al.*, 1955).

In static limit (Chew *et al.*, 1957), one obtains from equation (3.15)

$$\frac{d\sigma(E_\gamma, \theta)}{d\Omega} \propto 2 + 3\sin^2\theta \tag{3.20}$$

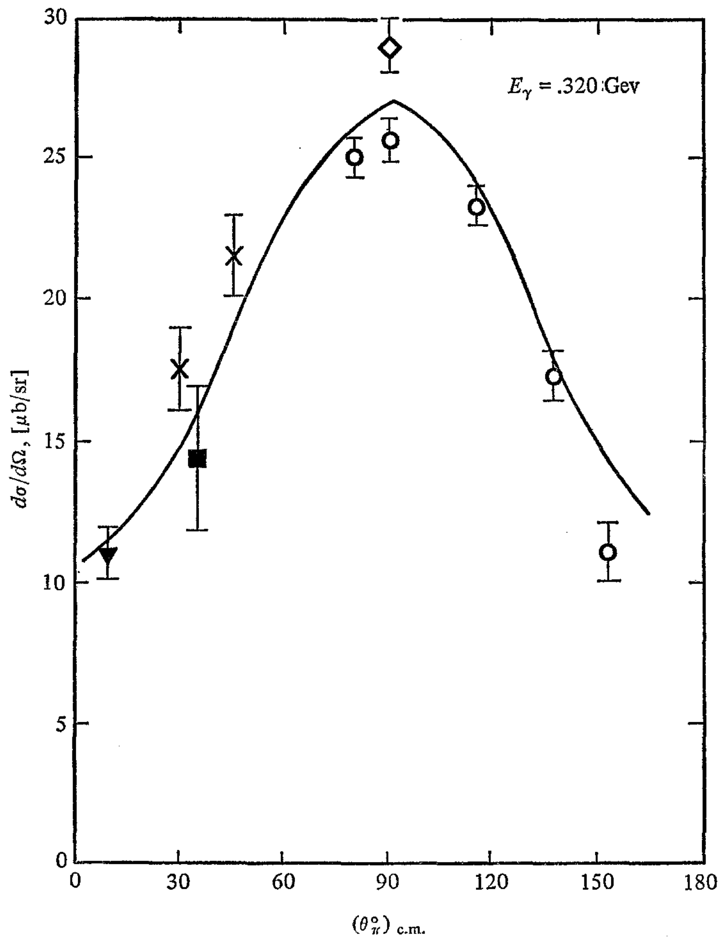

Figure 5.—Center-of-mass angular distributions of π^0 photoproduction at $E_\gamma = 0{\cdot}320$ Gev. Solid curve is the present theory.

The prediction of equation (3.20) is also consistent with the result of isobar model (Gourdin & Salin, 1963). Namely, for neutral pion photoproduction in the region of first resonance all angular distributions are roughly expressed by $2 + 3\sin^2\theta$. Comparing with experimental π^0 production data, as shown in Figs. 1 and 2, one sees that the first resonance is essentially dominated by the $M_{1+}^{(3)}$ transition and the agreement is seen to be good. The angular distributions at photon laboratory energies of 0·295, 0·300, 0·32 Gev are given from Fig. 3 to Fig. 5 and the agreements are quite satisfactory.

4. *Discussions and Conclusions*

On comparison with experiments in case of neutral pion photoproduction, the predictions of the present theory are found to be in good agreement. Above all, this formalism provides us with a simple model for photoproduction processes. It further verifies the conclusion that the large resonances appearing in total cross sections for pion-nucleon scattering and photopion productions are caused by the same resonant intermediate state of the same total energy of the whole system.

There are only two constants to be determined in this formalism, namely q_0 and Ω_0. Both of these constants have clear and definite physical meanings. The constant q_0 represents the mean energy of intermediate resonant states of the system. As mentioned in Section 3, for low energy π^0 production the proton is not excited and there is no charge exchange, thus the essential functions of the proton are assumed to provide the domain Ω_0 for the process and to conserve the momenta. Therefore, the proton energy is not included in the constant q_0. The active parts of the system in intermediate states are assumed to be localized within Ω_0. In concept, the present model is in this respect similar to the strong absorption model.

For neutral pion photoproduction the domain Ω_0 turns out to be $3{\cdot}88F$. It is very interesting to note that the extent of electromagnetic distribution in a proton is also nearly the size of Ω_0 (Hofstadter & Herman, 1961). This is believed to be the reason, which appears so clearly in the present formalism, that in low energy pion photoproduction the effects of electromagnetic structure of a nucleon are irrelevant. On the other hand, in case of vector meson or strange particle photoproduction, the domain Ω is expected to be much smaller and the effects of electromagnetic structure of a nucleon on the production processes are expected to be great.

Appendix

We display here the derivations of some of the restricting conditions on the form function. Now, let us recall that under an inhomogeneous Lorentz transformation the 4-coordinate transforms as

$$\delta x_\mu = \epsilon_\mu - \epsilon_{\mu\nu} x_\nu \tag{A1}$$

and a field transforms as

$$\delta\psi^\alpha(x) = \delta_0 \psi^\alpha(x) + \psi^\alpha_{,\lambda}(x)\,\delta x_\lambda + \epsilon_{\mu\nu} S^{\alpha\beta}_{\mu\nu} \psi^\beta(x) \tag{A2}$$

where the last two terms in (A2) are the infinitesimal transformations of the field induced by the transformation of (A1), while the first term in (A2) is due to a change in the functional form of $\psi^\alpha(x)$. The total change of action functional under an inhomogeneous Lorentz transformation can be written as

$$\delta A = \int_{\sigma_1}^{\sigma_2} \left\{ \frac{\partial \mathscr{L}(x)}{\partial \psi^\alpha(x)} \delta_0 \psi^\alpha(x) + \frac{\partial \mathscr{L}(x)}{\partial \psi^\alpha_{,\nu}(x)} \delta_0 \psi^\alpha_{,\nu}(x) + \frac{\partial \mathscr{L}(x)}{\partial x_\mu} \delta x_\mu \right\} d^4 x \quad \text{(A3)}$$

where σ_1 and σ_2 are space-like surfaces, σ_2 is later than σ_1. The integration is taken over a space-time domain bound by σ_1 and σ_2. For translational transformation only, i.e.,

$$\delta x_\mu = \epsilon_\mu$$
$$\epsilon_{\mu\nu} = 0$$
$$\delta \psi^\alpha(x) = 0$$

and

$$\delta_0 \psi^\alpha(x) = -\psi^\alpha_{,\nu}(x)\, \epsilon_\mu$$
$$\delta_0 \psi^\alpha_{,\nu}(x) = -\psi^\alpha_{,\mu\nu}(x)\, \epsilon_\mu$$

equation (A3) reduces immediately to

$$\delta_{\epsilon\mu} A = \epsilon_\mu \int_{\sigma_1}^{\sigma_2} \left(\frac{\partial \mathscr{L}(x)}{\partial x_\mu} \right) d^4 x \quad \text{(A4)}$$

and (A4) is valid for an arbitrary field. Generally, the Lagrangian functional can be expressed as

$$\mathscr{L}(x) = \sum_{n=1}^{N} n \int_{\sigma_1}^{\sigma_2} \cdots \int d^4 x' \ldots d^4 x^{n-1} \mathscr{L}^{(n)}(x, x', \ldots, x^{n-1}; \psi^\alpha(x), \psi^\alpha(x'), \ldots,$$
$$\times\, \psi^\alpha(x^{n-1}); \psi^\alpha_{,\nu}(x), \psi^\alpha_{,\nu}(x'), \ldots, \psi^\alpha_{,\nu}(x^{n-1})) \quad \text{(A5)}$$

where all integrations are taken over the same space-time domain bound by σ_1 and σ_2, and equation (A4) is rewritten as

$$\delta_{\epsilon\mu} A = \epsilon_\mu \sum_{n=1}^{N} \int_{\sigma_1}^{\sigma_2} \cdots \int \sum_{i=1}^{n} \left(\frac{\partial \mathscr{L}^{(n)}}{\partial x_\mu{}^i} \right) d^4 x \ldots d^4 x^{n-1} \quad \text{(A6)}$$

For translational invariance, i.e. $\delta_{\epsilon\mu} A = 0$, one obtains the necessary and sufficient condition

$$\sum_{i=1}^{n} \left(\frac{\partial}{\partial x_\mu{}^i} \right) \mathscr{L}^{(n)} = 0 \quad \text{(A7)}$$

In case the non-interacting part of the Lagrangian functional is not an explicit function of $x_\mu{}^i$, and for a non-local interaction the Lagrangian density can be expressed

$$\mathscr{L}^{(n)} = \mathscr{L}_0(\psi_1(x'), \ldots, \psi_n(x^n)) + \frac{\lambda}{n!} \sum_P M(x', \ldots, x^n)\, \psi_1(x') \ldots \psi_n(x^n) \quad \text{(A8)}$$

where $\sum_P$ indicates the sum over all permutations among $x', \ldots, x^n$ and the function $M(x', \ldots, x^n)$ is the form function for non-local interaction between the fields $\psi_1(x'), \ldots, \psi_n(x^n)$. Combining equations (A7) and (A8) one immediately obtains the restriction

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_\mu^{\,i}} M(x', \ldots, x^n) = 0 \tag{A9}$$

which is equation (4) in Section 2.

For rotational transformation of space-time only, one has

$$\begin{aligned} \epsilon_\mu &= 0 \\ \delta x_\mu &= -\epsilon_{\mu\nu} \chi_\nu \\ \delta \psi^\alpha(x) &= 0 \end{aligned}$$

and

$$\begin{aligned} \delta_0 \psi^\alpha(x) &= \epsilon_{\mu\nu}(\psi^\alpha_{,\mu}(x)\, x_\nu - S^{\alpha\beta}_{\mu\nu}\, \psi^\beta(x)) \\ \delta_0 \psi^\alpha_{,\lambda}(x) &= \epsilon_{\mu\nu}(\psi^\alpha_{,\mu\lambda}(x)\, x_\nu + \psi^\alpha_{,\mu}(x)\, \delta_{\lambda\nu} - S^{\alpha\beta}_{\mu\nu}\, \psi^\beta_{,\lambda}(x)) \end{aligned}$$

and by using the Lagrange equation,

$$\frac{\partial \mathscr{L}}{\partial \psi^\alpha(x)} - \frac{\partial}{\partial x_\lambda} \frac{\partial \mathscr{L}}{\partial \psi^\alpha_{,\lambda}(x)} = 0$$

the total variation of action functional can be expressed as

$$\delta_{\epsilon_{\mu\nu}} A = \frac{\epsilon_{\mu\nu}}{2} \int_{\sigma_1}^{\sigma_2} (T_{\mu\nu} - T_{\nu\mu})\, d^4 x + \frac{\epsilon_{\mu\nu}}{2} \sum_{n=1}^{N} \int_{\sigma_1}^{\sigma_2} \cdots \int \sum_{i=1}^{n}$$

$$\times \left\{ x_\mu^{\,i} \left(\frac{\partial \mathscr{L}^{(n)}}{\partial x_\nu^{\,i}} \right) - x_\nu^{\,i} \left(\frac{\partial \mathscr{L}^{(n)}}{\partial x_\mu^{\,i}} \right) \right\} d^4 x\, d^4 x' \ldots d^4 x^n \tag{A10}$$

where we have made use of the following relations

$$T_{\nu\mu} = \delta_{\nu\mu} \mathscr{L} - \frac{\partial \mathscr{L}}{\partial \psi^\alpha_{,\nu}} \psi^\alpha_{,\mu} - \frac{\partial f_{\lambda\nu\mu}(x)}{\partial x_\lambda}$$

$$\epsilon_{\nu\mu} f_{\lambda\nu\mu} = \frac{\partial \mathscr{L}}{\partial \psi^\alpha_{,\lambda}} \epsilon_{\nu\mu} S^{\alpha\beta}_{\nu\mu} \psi^\beta(x)$$

and

$$\begin{aligned} S^{\alpha\beta}_{\nu\mu} &= -S^{\alpha\beta}_{\mu\nu} \\ \epsilon_{\nu\mu} &= -\epsilon_{\mu\nu} \end{aligned}$$

It can be shown that in case of local-interaction theory the energy-momentum tensor $T_{\nu\mu}$ itself is symmetric. Now for simplicity, we assumed that for non-local interaction a kind of integral symmetry of the energy-momentum tensor will be satisfied, i.e.

$$\int_{\sigma_1}^{\sigma_2} T_{\mu\nu}\, d^4 x = \int_{\sigma_1}^{\sigma_2} T_{\nu\mu}\, d^4 x \tag{A11}$$

Then combining equations (A11) and (A10) and requiring the invariance of the total action functional, one obtains

$$\sum_{i=1}^{n} \left(x_{\mu}{}^{i} \frac{\partial}{\partial x_{\nu}{}^{i}} - x_{\nu}{}^{i} \frac{\partial}{\partial x_{\mu}{}^{i}} \right) \mathscr{L}^{(n)} = 0 \tag{A12}$$

Substituting equation (A8) into equation (A12), one has

$$\sum_{i=1}^{n} \left(x_{\mu}{}^{i} \frac{\partial}{\partial x_{\nu}{}^{i}} - x_{\nu}{}^{i} \frac{\partial}{\partial x_{\mu}{}^{i}} \right) M(x', \ldots, x^{n}) = 0 \tag{A13}$$

which is equation (5) in Section 2.

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